COULOMB AND CORE/HALO CORRECTIONS TO BOSE-EINSTEIN N-PARTICLE CORRELATIONS

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We report on a systematic treatment of Coulomb corrections for 3- and n-particle (Bose-Einstein) correlations, leading to a removal of a 100 % systematic error from earlier Coulomb corrections of 5-particle Bose-Einstein correlations in heavy ion collisions. These results are generalized to include strong final state interactions and core-halo effects.

1 Introduction

One of the most important tasks of high energy heavy-ion studies is to prove the existence of the elusive quark-gluon plasma and to study the properties of this predicted new state of matter[1]. Hanbury-Brown Twiss (HBT) interferometry[2] of identical particles has become an important tool as it can be used to measure the evolving geometry of the interaction region, see refs.[3,4,5,6,7] for some of the recent reviews of this rapidly expanding field. The quantitative interpretation of the HBT-results depends critically on the understanding of the rôle of the Coulomb interaction between the selected particles. However, Coulomb interactions between three (or more) charged particles are notoriously difficult to handle, as evidenced by the many decades of research in atomic physics aimed at obtaining solutions of the three-body Coulomb scattering problem which are accurate and possess a wide range of applicability.

Starting from a three-body Coulomb wave function which is at least ex-

act for sufficiently large inter-particle separations, we determined the effect of Coulomb final state interactions on the three-particle Bose-Einstein correlation function of similarly charged particles in ref.[9]. We estimated numerically that the familiar Riverside approximation is not precise enough to determine the three-body Coulomb correction factor in the correlation function if the characteristic HBT radius parameter is between 5 and 10 fm, which is the range of interest in high-energy heavy ion physics.

We then generalized the method of Coulomb wave function corrections from three to n charged particles in ref.[10]. Here, we re-formulate this work so as to be able to include the possible effects of i) additional strong final state interactions and ii) core-halo model corrections.

2 Coulomb and final state interaction effects in n-particle correlations

The n-particle Bose-Einstein correlation function is defined as

$$C_n(\mathbf{k}_1, \cdots, \mathbf{k}_n) = \frac{N_n(\mathbf{k}_1, \cdots, \mathbf{k}_n)}{N_1(\mathbf{k}_1) \cdots N_1(\mathbf{k}_n)}, \tag{1}$$

where $N_n(\mathbf{k}_1, \cdots, \mathbf{k}_n)$ is the n-particle, and $N_1(\mathbf{k}_i)$ the single-particle inclusive invariant momentum distribution. The three-momentum vector of particle i is denoted by $\mathbf{k}_i$. It is quite remarkable that this complicated object, that carries quantum mechanical information on the phase-space distribution of particle production as well as on possible partial coherence of the source, can be expressed in a relatively simple, straight-forward manner both in the analytically solvable pion-laser model of refs.[11,12,13] as well as in the generic boosted-current formalism of Gyulassy and Padula[14] as

$$C_n(\mathbf{k}_1, \cdots, \mathbf{k}_n) = \frac{\sum_{\sigma^{(n)}} \prod_{i=1}^{n} G(\mathbf{k}_i, \mathbf{k}_{\sigma_i})}{\prod_{i=1}^{n} G(\mathbf{k}_i, \mathbf{k}_i)}, \tag{2}$$

where $\sigma^{(n)}$ stands for the set of permutations of indices $(1, 2, \cdots, n)$ and σ_i denotes that element which replaces element i in a given permutation from the set of $\sigma^{(n)}$. Regardless of the details of the two different derivations

$$G(\mathbf{k}_i, \mathbf{k}_j) = \langle a^\dagger(\mathbf{k}_i) a(\mathbf{k}_j) \rangle \tag{3}$$

stands for the expectation value of $a^\dagger(\mathbf{k}_i) a(\mathbf{k}_j)$.

In the relativistic Wigner-function formalism, in the plane wave approximation $G(\mathbf{k}_1, \mathbf{k}_2)$ can be rewritten as

$$G(\mathbf{k}_1, \mathbf{k}_2) = \int d^4x\, S(x, K_{12}) \exp(iq_{12} \cdot x) \tag{4}$$

$$K_{12} = 0.5(k_1 + k_2), \qquad q_{12} \;=\; k_1 - k_2, \tag{5}$$

where a four-vector notation is introduced, $k_i = (\sqrt{m_i^2 + \mathbf{k}_i^2}, \mathbf{k}_i)$, and $a \cdot b$ stands for the inner product of four-vectors. Due to the mass-shell constraints $E_{\mathbf{k}_i} = \sqrt{m_i^2 + \mathbf{k}_i^2}$, G depends only on 6 independent momentum components. In any given frame, the boost-invariant decomposition of Eq. (4) can be rewritten in the following, seemingly not invariant form:

$$G(\mathbf{k}_1, \mathbf{k}_2) = \int d^3\mathbf{x}\; S_{\mathbf{K}_{12}}(\mathbf{x})\; \exp(-i\mathbf{q}_{12}\mathbf{x}), \tag{6}$$

$$S_{\mathbf{K}_{12}}(\mathbf{x}) = \int dt\, \exp(i\boldsymbol{\beta}_{K_{12}}\mathbf{q}_{12}t)\; S(\mathbf{x}, t, K_{12}), \tag{7}$$

$$\boldsymbol{\beta}_{K_{12}} = (\mathbf{k}_1 + \mathbf{k}_2)/(E_1 + E_2). \tag{8}$$

If n particles are emitted with similar momenta so that their n-particle Bose-Einstein correlation functions may be non-trivial, Eqs. (2,3) will form the basis for evaluation of the Coulomb and strong final state interaction effects on the observables. On this level, all the correlations are build up from correlations of pairs of particles.

In order to treat the Coulomb corrections to the n-particle correlation function exactly, knowledge of the n-body Coulomb scattering wave function is required. We restrict ourselves to the case that the transverse momenta of all n particles in the final state in their common center of mass are small enough to make a nonrelativistic approach sensible. Hence the problem consists in finding the solution of the n-charged particle Schrödinger equation when all n particles are in the continuum.

Consider n distinguishable particles with masses m_i and charges e_i, $i = 1, 2, \cdots, n$. Let $\mathbf{x}_i$ denote the coordinate (three-)vector of particle i. Then $\mathbf{r}_{ij} = \mathbf{x}_i - \mathbf{x}_j$ is the relative coordinate between particles i and j, and $\mathbf{k}_{ij} = (m_j\mathbf{k}_i - m_i\mathbf{k}_j)/(m_i + m_j)$ the canonically conjugate relative momentum.

The n-particle Schrödinger equation reads as

$$\left\{ H_0 + \sum_{i<j=1}^{n} V_{ij} - E \right\} \Psi^{(+)}_{\mathbf{k}_1 \cdots \mathbf{k}_n}(\mathbf{x}_1, \cdots, \mathbf{x}_n) = 0, \tag{9}$$

where

$$E = \sum_{i=1}^{n} \frac{\mathbf{k}_i^2}{2m_i} > 0 \tag{10}$$

is the total kinetic energy for n particles in the continuum. H_0 is the free Hamilton operator and

$$V_{ij}(\mathbf{r}_{ij}) = V_{ij}^S(\mathbf{r}_{ij}) + V_{ij}^C(\mathbf{r}_{ij}) \tag{11}$$

the interaction between particles i and j, consisting of a short-range strong potential V_{ij}^S and the long-range Coulomb interaction, $V_{ij}^C(\mathbf{r}_{ij}) = e_i e_j / \mid \mathbf{r}_{ij} \mid$.

Although for $n = 3$ an exact numerical solution of the Schrödinger equation (9) with $V_{ij} = V_{ij}^C$ has been achieved recently for $E > 0$[16], because of its complexitiy the resulting wave function can not easily be used for general purposes. In addition, for $n > 3$ a numerical solution is beyond present means. For a brief discussion of the related difficulties see[17]. This means that for practical purposes analytic, even if only approximate, n-charged particle wave functions are still needed.

For $n = 3$ such wave functions are available in the form of the explicit solutions of the Schrödinger equation in all asymptotic regions of the three-particle configuration space[18,19]. The simplest form, which is also easily generalized to arbitrary particle numbers, applies to the (dominant) asymptotic region conventionally denoted by Ω_0 and characterized by the fact that - roughly speaking - all three inter-particle distances become uniformly large[19]. In the final states of heavy-ion reactions, where a large number of charged particle tracks appear, the mutual, macroscopically large separation of tracks is one of the criteria of a clean measurement. This suggests that in order to study Coulomb effects on n-body correlation functions, knowledge of the wave function in $\Omega_0^{(n)}$, the region in n-particle configuration space where all interparticle distances become uniformly large, i.e., $\mid \mathbf{r}_{ij} \mid \to \infty$ for all values of (ij), may be sufficient. Hence, an appropriate, approximate n-particle Coulomb scattering wave function is

$$\Psi^{(+)}_{\mathbf{k}_1,\cdots,\mathbf{k}_n}(\mathbf{x}_1, \cdots, \mathbf{x}_n) \;\sim\; \prod_{i<j=1}^{n} \psi^{C(+)}_{\mathbf{k}_{ij}}(\mathbf{r}_{ij}). \tag{12}$$

Here, $\psi^{C(+)}_{\mathbf{k}_{ij}}(\mathbf{r}_{ij})$ is the continuum solution of the two-body Coulomb Schrödinger equation (with a reduced mass of $\mu_{ij} = m_i m_j/(m_i + m_j)$):

$$\left\{ -\frac{\Delta_{\mathbf{r}_{ij}}}{2\mu_{ij}} + V_{ij}^C(\mathbf{r}_{ij}) - \frac{\mathbf{k}_{ij}^2}{2\mu_{ij}} \right\} \psi^{C(+)}_{\mathbf{k}_{ij}}(\mathbf{r}_{ij}) = 0, \tag{13}$$

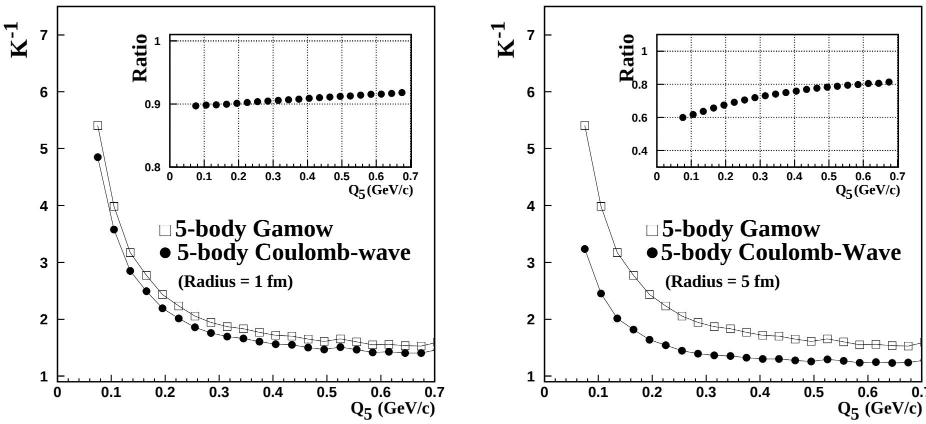

Figure 1. Coulomb wave function correction (filled circles) as compared to the less substantiated Gamow correction (squares) for 5-particle Bose-Einstein correlations for a source sizes $R = 1$ fm (left panel) and $R = 5$ fm (right panel). The inset shows the ratio of these two correction factors.

describing the relative motion of the two particles i and j with energy $\mathbf{k}_{ij}^2/2\mu_{ij}$. The explicit solution is

$$\psi^{C(+)}_{\mathbf{k}_{ij}}(\mathbf{r}_{ij}) = N_{ij}\, e^{i\mathbf{k}_{ij}\mathbf{r}_{ij}} F[-i\eta_{ij}, 1; i(|\,\mathbf{k}_{ij}\,||\,\mathbf{r}_{ij}\,| - \mathbf{k}_{ij}\mathbf{r}_{ij})], \tag{14}$$

with $N_{ij} = e^{-\pi\eta_{ij}/2}\,\Gamma(1 + i\eta_{ij})$, and $\eta_{ij} = e_i e_j \mu_{ij}/\mid \mathbf{r}_{ij} \mid$ being the Coulomb parameter. $F[a, b; x]$ is the confluent hypergeometric function and $\Gamma(x)$ the Gamma function. The foundation of Eq. (12) in fundamental scattering theory and its expected range of validity are discussed in ref.[10].

Additional strong final state interactions can be taken into account in a straight-forward manner by substituting in Eq. (12) for the two-particle Coulomb wave functions $\psi^{C(+)}_{\mathbf{k}_{ij}}(\mathbf{r}_{ij})$ the solutions of the Schrödinger equation with the Coulomb plus strong potential,

$$\left\{ -\frac{\Delta_{\mathbf{r}_{ij}}}{2\mu_{ij}} + V^C_{ij}(\mathbf{r}_{ij}) + V^S_{ij}(\mathbf{r}_{ij}) - \frac{\mathbf{k}^2_{ij}}{2\mu_{ij}} \right\} \psi^{CS(+)}_{\mathbf{k}_{ij}}(\mathbf{r}_{ij}) = 0. \tag{15}$$

In this way a few dominant, strong interaction induced phase-shifts can be taken into account for the pair under consideration. On the level of the two-particle correlation function, a similar technique has already been applied in ref.[20].

3 Application to high-energy heavy-ion and particle collisions

The correlation function measuring the enhanced probability for emission of n identical Bose particles is given by Eq. (1). This correlation function is usually, due to meager statistics, only measured as a function of the Lorentz invariant Q_n, defined by the relation

$$Q_n^2 = \sum_{i<j=1}^{n} (k_i - k_j)^2. \tag{16}$$

We can now calculate the Coulomb effects on the n-particle correlation function using

$$K_{Coulomb}(Q_n) = \frac{\int \prod_{i=1}^{n} d^3\mathbf{x_i}\rho(\mathbf{x_i}) \left|\Psi^{(+)\mathcal{S}}_{\mathbf{k_1}\cdots\mathbf{k}_n}(\mathbf{x}_1,\cdots,\mathbf{x}_n)\right|^2}{\int \prod_{i=1}^{n} d^3\mathbf{x_i}\rho(\mathbf{x_i}) \left|\Psi^{(0)\mathcal{S}}_{\mathbf{k_1}\cdots\mathbf{k}_n}(\mathbf{x}_1,\cdots,\mathbf{x}_n)\right|^2}. \tag{17}$$

Here, $\rho(\mathbf{x_i})$ is the density distribution of the source for particle i (normalized to the total number of particles), taken as a Gaussian distribution of width R in all three spatial directions and $\Psi^{(0)}_{\mathbf{k_1}\cdots\mathbf{k}_n}(\mathbf{x}_1,\cdots,\mathbf{x}_n) \sim \prod_{i<j=1}^{n} e^{i\mathbf{k}_{ij}\mathbf{r}_{ij}}$ is the n-body wave function without any final state interaction. The superscript $\mathcal{S}$ indicates appropriate symmetrisation for identical particles. This formulation makes it possible to extract information on the source size R, and to compare this value with that extracted by means of a generalized n-particle Gamow approximation via $K^{(G)}_{Coulomb}(Q_n) = \prod_{i<j=1}^{n} G_{ij}$, where $G_{ij} =\mid N_{ij} \mid^2$.

Pion n-tuples were sampled randomly[9,10] from the NA44 data sample of three pion events produced in S-Pb collisions at CERN[21]. Fig. 1 demonstrates that in case of a characteristic effective source size of 1 fm, the difference between the n-particle Gamow and the Coulomb wave function corrections are smaller than 10 % for $n = 5$ particles. However, with increasing number of particles and/or with increasing effective source sizes, this difference increases dramatically. For instance, for 5 particles the naive generalized Gamow method overestimates the Coulomb correction as compared to the much better substantiated Coulomb wave function integration method by a factor of 2.

4 Core-halo corrections

The results given in the previous section have to be corrected for the effects that may arise from the existence of a halo of long-lived hadronic

resonances[22,23]. In this case, the effective source function $\rho(x)$ has two components, one pertinent to a smaller region corresponding to the core of the interactions and one to a larger region that corresponds to the (unresolvable) halo:

$$\rho(x) = \rho_c(x) + \rho_h(x) \tag{18}$$

$$\langle n \rangle = \langle n_c \rangle + \langle n_h \rangle \tag{19}$$

$$\langle n_i \rangle = \int dx \rho_i(x), \quad i = c, h. \tag{20}$$

The effective intercept parameter of the two-particle Bose-Einstein correlation function reads as

$$\lambda_* = (\langle n_c \rangle / \langle n \rangle)^2 \tag{21}$$

if the core fraction is independent of the relative momentum. If the core-halo model is applicable to a given data set, the radius R_h of the halo does not matter as long as it is big enough. For instance, it can be a Gaussian with a radius of 20-40 fm; in any case $R_h >> \hbar/Q_{min}$, the inverse of the two-particle relative momentum resolution.

5 Summary

In this contribution we reviewed the state-of-art method for treating Coulomb corrections to 3- and n-body correlation functions. For small effective source sizes of about 1 fm, the pairwise product of Gamow factors gives a good approximation for the n-body Coulomb correction factor. However, for source sizes typical in high energy heavy ion experiments, a more substantiated calculation based on the pairwise product of relative Coulomb wave functions is required in order to remove a possible 100 % systematic error from this correction.

We have also generalized this correction method for the inclusion of strong final state interactions and pointed out that the Coulomb corrections have to be performed self-consistently with the core-halo corrections; in particular the effective intercept parameter of the two-particle Bose-Einstein correlation function has to be taken into account when evaluating the 3- or n-body Coulomb corrections.

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